CALCULATION SHEET

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Sheet No. 1			
Project No.			
Calc. By	Date	Rev. A1	
Checked By	Date		

Project Title:

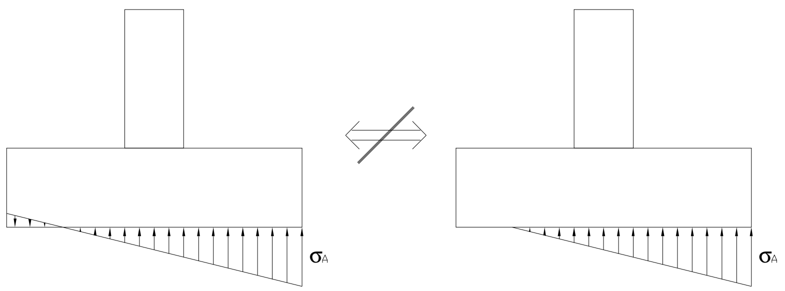
Subject/Feature: FOUNDATION 3D FINITE ELEMENT ANALYSIS – RESULTS VALIDATION

General Index

A. INTRODUCTION (NARRATIVE DESCRIPTION)

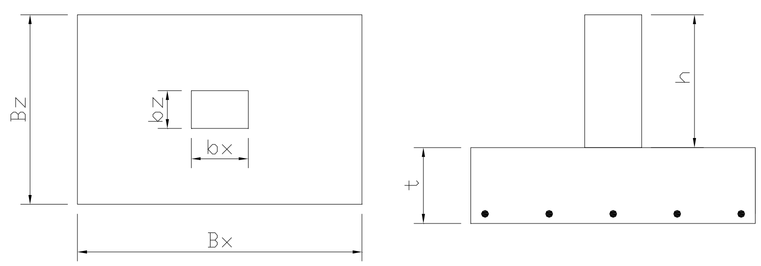
A.1. Objective

The objective of this calculation is to validate the results of the foundations calculations using 3D finite element analysis performed in STAAD.Pro. Such a complex calculation is justified when a foundation is biaxial loaded with high magnitude forces, such that we have large eccentricities on both directions and consequently uplift at one end of the foundation. A common mistake in the design is to simply ignore the negative pressures. The two situations presented below are not equivalent.



There are two correct solutions in the design of a foundation. The classical approach is to increase the size of the footing until the uplift is 0, but this may lead to overdimensioning. The most economical way to calculate a foundation, is to consider the uplift.

Quick comparative calculation:



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Allowable soil pressure:

$p_{allowable} := 150 \frac{kN}{m^2}$

Loading

$V_y := 30kN$ $H_z := 0kN$ $H_x := 6kN$ $M_z := 5kN \cdot m$ $M_x := 0kN \cdot m$

Foundation dimensions

$B_x := 1m$ $b_x := 0.4m$ $t := 0.4m$

$B_z := 1m$ $b_z := 0.4m$ $h := 1m$

Soil pressures (at corners)

Foundation weight

$W_p := (b_x \cdot b_z \cdot h + B_x \cdot B_z \cdot t) 26 \frac{kN}{m^3}$ $W_p = 14.56kN$

Forces and moments translated at the base of the foundation

$V_{y.b} := V_y + W_p$ $V_{y.b} = 44.56kN$

$M_{z.b} := M_z + H_x \cdot (t + h)$ $M_{z.b} = 13.4kN \cdot m$

$M_{x.b} := M_x + H_z \cdot (t + h)$ $M_{x.b} = 0 kN \cdot m$

$p_1 := \frac{V_{y.b}}{B_x \cdot B_z} + \frac{6 \cdot M_{z.b}}{B_z \cdot B_x^2} + \frac{6 \cdot M_{x.b}}{B_x \cdot B_z^2}$ $p_1 = 124.96 \frac{kN}{m^2}$

$p_2 := \frac{V_{y.b}}{B_x \cdot B_z} + \frac{6 \cdot M_{z.b}}{B_z \cdot B_x^2} + \frac{-6 \cdot M_{x.b}}{B_x \cdot B_z^2}$ $p_2 = 124.96 \frac{kN}{m^2}$

$p_3 := \frac{V_{y.b}}{B_x \cdot B_z} + \frac{-6 \cdot M_{z.b}}{B_z \cdot B_x^2} + \frac{-6 \cdot M_{x.b}}{B_x \cdot B_z^2}$ $p_3 = -35.84 \frac{kN}{m^2}$

$p_4 := \frac{V_{y.b}}{B_x \cdot B_z} + \frac{-6 \cdot M_{z.b}}{B_z \cdot B_x^2} + \frac{6 \cdot M_{x.b}}{B_x \cdot B_z^2}$ $p_4 = -35.84 \frac{kN}{m^2}$

Eccentricity:

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Eccentricity:

$e_x := \frac{M_{z.b}}{V_{y.b}}$ $\quad e_x = 0.301m$

Classical approach vs. actual soil pressure determination considering uplift.

Classical approach:

Redimensioning of the foundation:

New foundation dimensions

$B_{x.new} := 1m \quad B_{z.new} := 4.5m$

New foundation weight

$W_{p.new} := \left(b_x \cdot b_z \cdot h + B_{x.new} \cdot B_{z.new} \cdot t\right) 26 \frac{kN}{m^3}$

$W_{p.new} = 50.96kN$

Forces and moments
at the base of the foundation:

$V_{y.b.new} := V_y + W_{p.new} \quad V_{y.b.new} = 80.96kN$

$M_{z.b} = 13.4kN \cdot m$

$M_{x.b} = 0kN \cdot m$

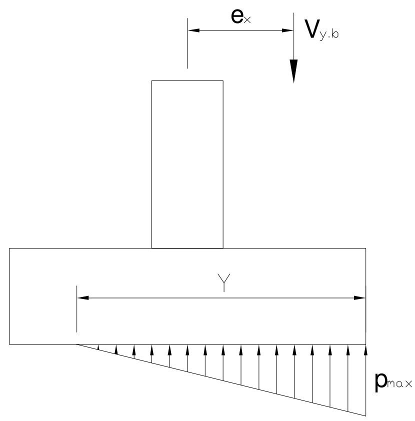
$$p_{1.new} := \frac{V_{y.b.new}}{B_{x.new} \cdot B_{z.new}} + \frac{6 \cdot M_{z.b}}{B_{z.new} \cdot B_{x.new}^2} + \frac{6 \cdot M_{x.b}}{B_{x.new} \cdot B_{z.new}^2}$$

$p_{1.new} = 35.858 \frac{kN}{m^2}$

Calculation considering the uplift:

Guess values:

$p_{max} := 150 \frac{kN}{m^2} \quad Y := 0.8m$



Given $\frac{Y \cdot p_{max} \cdot B_z}{2} = V_{y.b}$

$e_x + \frac{Y}{3} = \frac{B_x}{2}$

Solutions:

$Y = 0.598m$

$p_{max} = 149.069 \frac{kN}{m^2}$

$$p_{2.new} := \frac{V_{y.b.new}}{B_{x.new} \cdot B_{z.new}} + \frac{6 \cdot M_{z.b}}{B_{z.new} \cdot B_{x.new}^{2}} + \frac{-6 \cdot M_{x.b}}{B_{x.new} \cdot B_{z.new}^{2}}$$

$$p_{2.new} = 35.858 \frac{kN}{m^2}$$

$$p_{3.new} := \frac{V_{y.b.new}}{B_{x.new} \cdot B_{z.new}} + \frac{-6 \cdot M_{z.b}}{B_{z.new} \cdot B_{x.new}^{2}} + \frac{-6 \cdot M_{x.b}}{B_{x.new} \cdot B_{z.new}^{2}}$$

$$p_{3.new} = 0.124 \frac{kN}{m^2}$$

$$p_{4.new} := \frac{V_{y.b.new}}{B_{x.new} \cdot B_{z.new}} + \frac{-6 \cdot M_{z.b}}{B_{z.new} \cdot B_{x.new}^{2}} + \frac{6 \cdot M_{x.b}}{B_{x.new} \cdot B_{z.new}^{2}}$$

$$p_{4.new} = 0.124 \frac{kN}{m^2}$$

Difference in weight between the two foundations:

$$Diff := \frac{W_{p.new}}{W_p} \qquad Diff = 350\%$$

So, the calculation of the foundation with uplift is quite simple if the loads acting longitudinally (H_x & M_z) are not simultaneous with those acting transversally (H_z & M_x).

In this case a more complex approach is necessary.

A.2. Concept

As mentioned before, a 3D finite element model composed of solids supported by compression only springs is used in the STAAD analysis. The STAAD input file is generated in Microsoft Excel. Since in STAAD moments cannot be applied to solids, the moments are taken into consideration as a couple of forces. For acceptable result accuracy the solid elements should not be larger than 0.15x0.15x0.2 (l_x x l_z x h) and l_x and l_z should be comparable in dimensions.

B. RESULTS EVALUATION

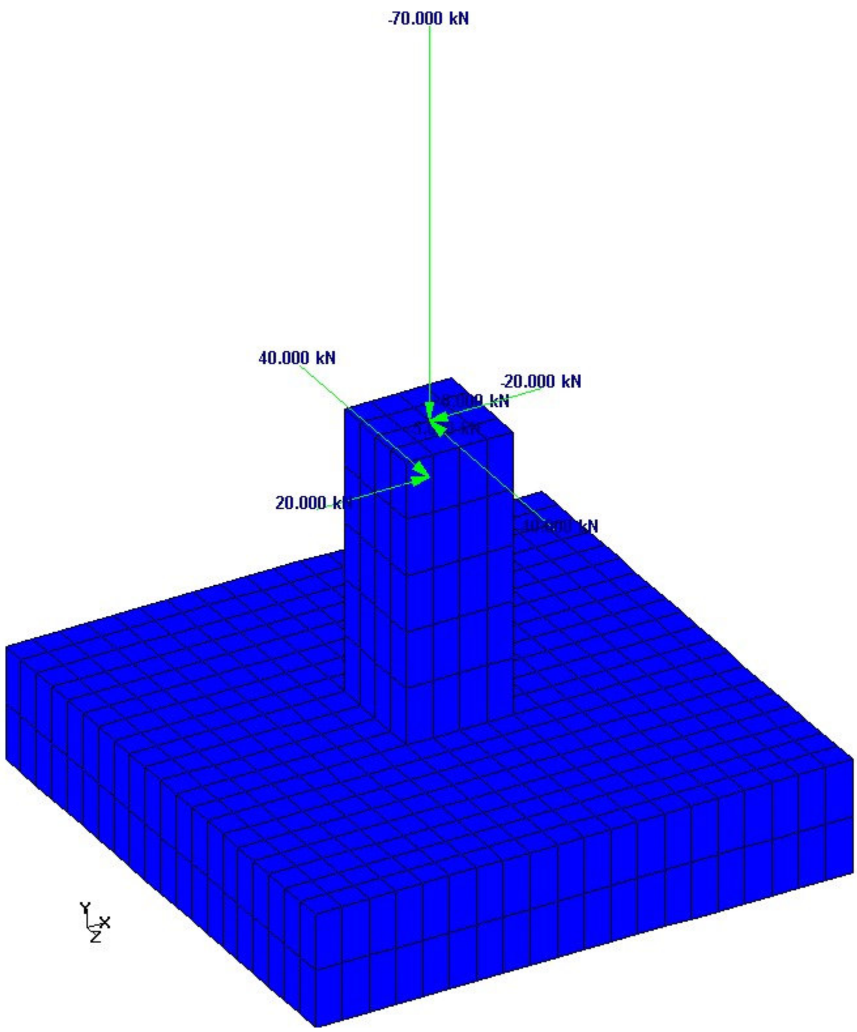
In order to evaluate the validity of the results three calculations were performed, and their results were checked against mathcad handcalcs.

B.1. MODEL NO. 1

B.1.1. Dimensions and loading

Lenght: Bx = 2 m Width: Bz =2 m

Height: H = 1.5 m Footing thickness: t = 0.4 m



B.1.2. Mathcad spreadsheet

Electronic file:

Check1.mcd

Staad check no 1.

$kN := 1000N$

Loading

$V_y := 70kN$ $H_z := 5kN$ $H_x := 8kN$ $M_z := 4kN \cdot m$ $M_x := 8kN \cdot m$

Foundation dimensions

$B_x := 2m$ $b_x := 0.4m$ $t := 0.4m$

$B_z := 2m$ $b_z := 0.4m$ $h := 1m$

Soil pressures (at corners)

Foundation weight

$W_p := (b_x \cdot b_z \cdot h + B_x \cdot B_z \cdot t) 26 \frac{kN}{m^3}$ $W_p = 45.76kN$

Forces and moments at the base of the foundation:

$V_{y.b} := V_y + W_p$ $V_{y.b} = 115.76kN$

$M_{z.b} := M_z + H_x \cdot (t + h)$ $M_{z.b} = 15.2kN \cdot m$

$M_{x.b} := M_x + H_z \cdot (t + h)$ $M_{x.b} = 15kN \cdot m$

$$p_1 := \frac{V_{y.b}}{B_z \cdot B_z} + \frac{6 \cdot M_{z.b}}{B_z \cdot B_x^2} + \frac{6 \cdot M_{x.b}}{B_x \cdot B_z^2} \qquad p_1 = 51.59 \frac{kN}{m^2}$$

$$p_2 := \frac{V_{y.b}}{B_z \cdot B_z} + \frac{6 \cdot M_{z.b}}{B_z \cdot B_x^2} + \frac{-6 \cdot M_{x.b}}{B_x \cdot B_z^2} \qquad p_2 = 29.09 \frac{kN}{m^2}$$

$$p_3 := \frac{V_{y.b}}{B_z \cdot B_z} + \frac{-6 \cdot M_{z.b}}{B_z \cdot B_x^2} + \frac{-6 \cdot M_{x.b}}{B_x \cdot B_z^2} \qquad p_3 = 6.29 \frac{kN}{m^2}$$

$$p_4 := \frac{V_{y.b}}{B_z \cdot B_z} + \frac{-6 \cdot M_{z.b}}{B_z \cdot B_x^2} + \frac{6 \cdot M_{x.b}}{B_x \cdot B_z^2} \qquad p_4 = 28.79 \frac{kN}{m^2}$$

B.1.3. STAAD Model

Electronic files:

Check1.std; Check1.xls

Results - Corner stresses:

Model 1

Node	L/C	Force-X kN	Force-Y kN	Force-Z kN
1	1	0.018	0.458	0.012
21	1	0.018	0.263	0.011
421	1	0.018	0.26	0.012
441	1	0.018	0.065	0.011

Support tributary area:

lx = 0.095 m ly = 0.095 m

A = 0.009025 m^2

Soil pressures @ corners

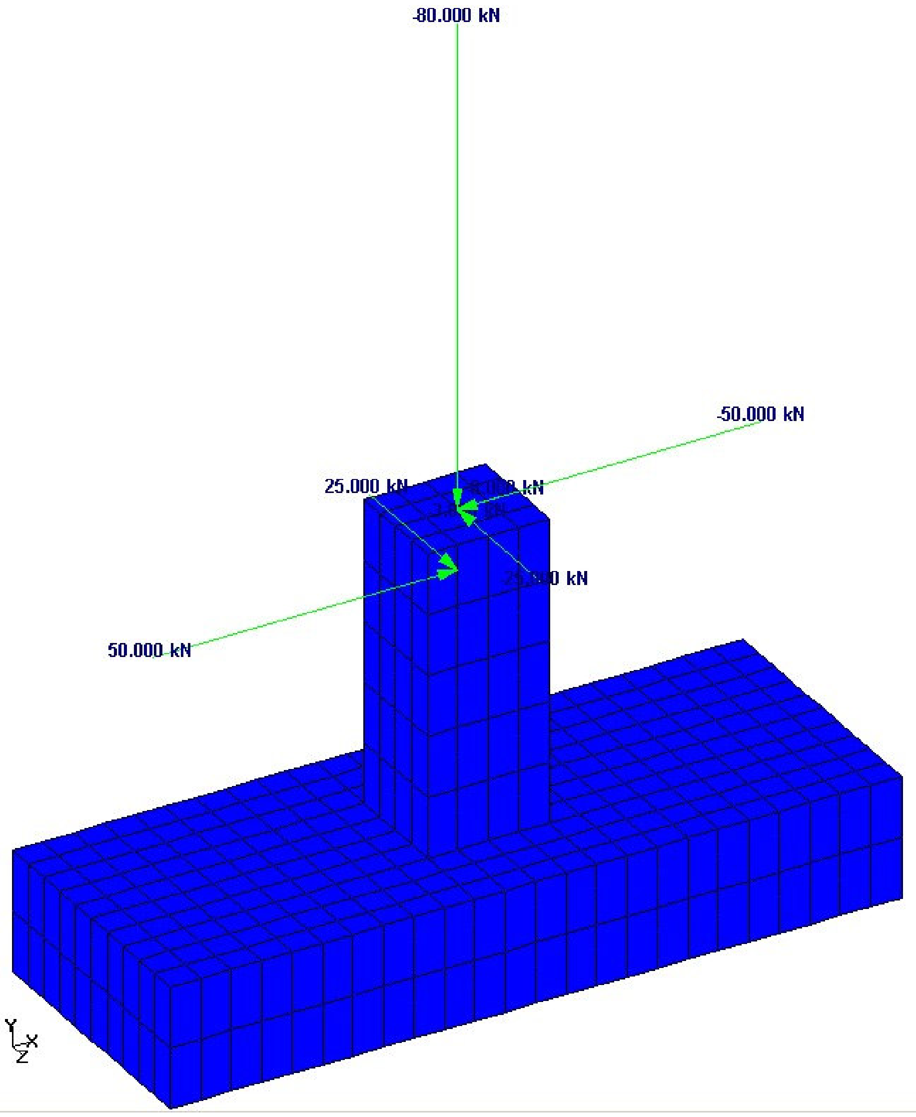
Node	L/C	STAAD P (kN/m^2)	MATHCAD P (kN/m^2)	DIFFERENCE %
1	1	50.75	51.59	1.63
21	1	29.14	29.09	-0.18
421	1	28.81	28.79	-0.07
441	1	7.20	6.29	-14.50

B.2. MODEL NO. 2

B.2.1. Dimensions and loading

Lenght: Bx = 2.5 m Width: Bz =1 m

Height: H = 1.4 m Footing thickness: t = 0.4 m



B.2.2. Mathcad spreadsheet

Electronic file:

Check2.mcd

Staad check no 2.

$kN := 1000N$

Loading

$V_y := 80.kN$ $H_z := 3kN$ $H_x := 8kN$ $M_z := 10kN \cdot m$ $M_x := 5kN \cdot m$

Foundation dimensions

$B_x := 2.5m$ $b_x := 0.4m$ $t := 0.4m$

$B_z := 1m$ $b_z := 0.4m$ $h := 1m$

Soil pressures (at corners)

Foundation weight

$W_p := \left(b_x \cdot b_z \cdot h + B_x \cdot B_z \cdot t\right) 26 \frac{kN}{m^3}$ $W_p = 30.16kN$

Forces and moments at the base of the foundation

$V_{y.b} := V_y + W_p$ $V_{y.b} = 110.16kN$

$M_{z.b} := M_z + H_x \cdot (t + h)$ $M_{z.b} = 21.2kN \cdot m$

$M_{x.b} := M_x + H_z \cdot (t + h)$ $M_{x.b} = 9.2kN \cdot m$

$$p_1 := \frac{V_{y.b}}{B_x \cdot B_z} + \frac{6 \cdot M_{z.b}}{B_z \cdot B_x^2} + \frac{6 \cdot M_{x.b}}{B_x \cdot B_z^2} \qquad p_1 = 86.496 \frac{kN}{m^2}$$

$$p_2 := \frac{V_{y.b}}{B_x \cdot B_z} + \frac{6 \cdot M_{z.b}}{B_z \cdot B_x^2} + \frac{-6 \cdot M_{x.b}}{B_x \cdot B_z^2} \qquad p_2 = 42.336 \frac{kN}{m^2}$$

$$p_3 := \frac{V_{y.b}}{B_x \cdot B_z} + \frac{-6 \cdot M_{z.b}}{B_z \cdot B_x^2} + \frac{-6 \cdot M_{x.b}}{B_x \cdot B_z^2} \qquad p_3 = 1.632 \frac{kN}{m^2}$$

$$p_4 := \frac{V_{y.b}}{B_x \cdot B_z} + \frac{-6 \cdot M_{z.b}}{B_z \cdot B_x^2} + \frac{6 \cdot M_{x.b}}{B_x \cdot B_z^2} \qquad p_4 = 45.792 \frac{kN}{m^2}$$

B.2.3. STAAD Model

Electronic files:

Check2.std; Check2.xls

Results - Corner stresses:

Model 2

Node	L/C	Force-X kN	Force-Y kN	Force-Z kN
1	1	0.03	0.762	0.011
11	1	0.03	0.391	0.011
265	1	0.028	0.407	0.011
275	1	0.028	0.035	0.011

Support tributary area:

lx = 0.1 m   ly = 0.09 m

A = 0.009 m^2

Soil pressures @ corners

Node	L/C	STAAD P (kN/m^2)	MATHCAD P (kN/m^2)	DIFFERENCE %
1	1	84.67	86.5	2.12
11	1	43.44	42.34	-2.61
265	1	45.22	45.79	1.24
275	1	3.89	1.63	-138.58

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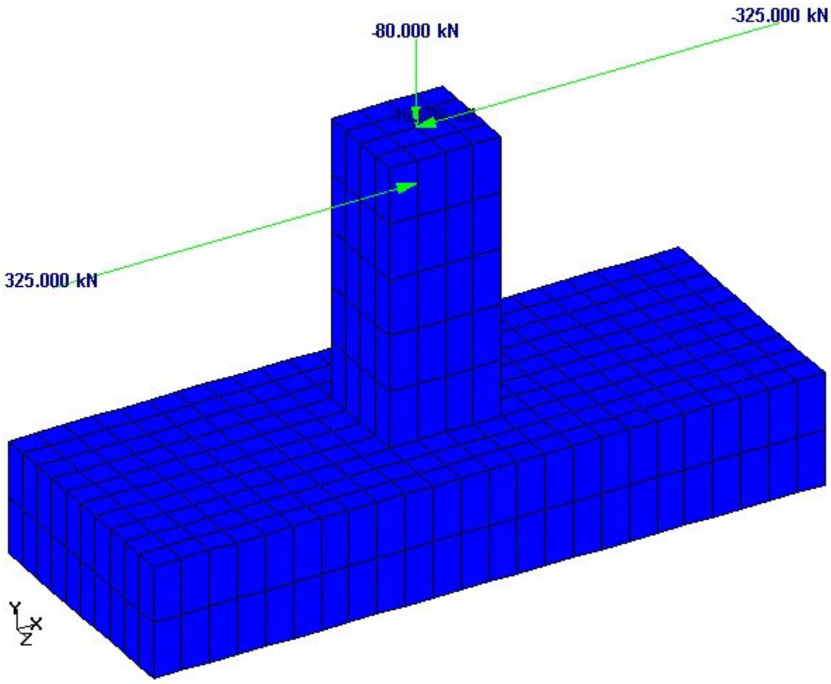
B.3. MODEL NO. 3

B.3.1. Dimensions and loading

Lenght: Bx = 2.5 m Width: Bz =1 m

Height: H = 1.4 m Footing thickness: t = 0.4 m



B.3.2. Mathcad spreadsheet

Electronic file:

Check3.mcd

Staad check no 3.

$kN := 1000N$

Loading

$V_y := 80kN$ $H_z := 0kN$ $H_x := 15kN$ $M_z := 65kN \cdot m$ $M_x := 0kN \cdot m$

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Foundation dimensions

$B_x := 2.5m$ $\quad b_x := 0.4m$ $\quad t := 0.4m$

$B_z := 1m$ $\quad b_z := 0.4m$ $\quad h := 1m$

Soil pressures (at corners)

Foundation weight

$$W_p := \left(b_x \cdot b_z \cdot h + B_x \cdot B_z \cdot t\right) 26 \frac{kN}{m^3} \qquad W_p = 30.16kN$$

Forces and moments at the base of the foundation

$$V_{y.b} := V_y + W_p \qquad V_{y.b} = 110.16kN$$

$$M_{z.b} := M_z + H_x \cdot (t + h) \qquad M_{z.b} = 86kN \cdot m$$

$$M_{x.b} := M_x + H_z \cdot (t + h) \qquad M_{x.b} = 0kN \cdot m$$

$$p_1 := \frac{V_{y.b}}{B_x \cdot B_z} + \frac{6 \cdot M_{z.b}}{B_z \cdot B_x^2} + \frac{6 \cdot M_{x.b}}{B_x \cdot B_z^2} \qquad p_1 = 126.624\frac{kN}{m^2}$$

$$p_2 := \frac{V_{y.b}}{B_x \cdot B_z} + \frac{6 \cdot M_{z.b}}{B_z \cdot B_x^2} + \frac{-6 \cdot M_{x.b}}{B_x \cdot B_z^2} \qquad p_2 = 126.624\frac{kN}{m^2}$$

$$p_3 := \frac{V_{y.b}}{B_x \cdot B_z} + \frac{-6 \cdot M_{z.b}}{B_z \cdot B_x^2} + \frac{-6 \cdot M_{x.b}}{B_x \cdot B_z^2} \qquad p_3 = -38.496\frac{kN}{m^2}$$

$$p_4 := \frac{V_{y.b}}{B_x \cdot B_z} + \frac{-6 \cdot M_{z.b}}{B_z \cdot B_x^2} + \frac{6 \cdot M_{x.b}}{B_x \cdot B_z^2} \qquad p_4 = -38.496\frac{kN}{m^2}$$

Eccentricity:

$$e_x := \frac{M_{z.b}}{V_{y.b}} \qquad e_x = 0.781m$$

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Guess values:

$p_{max} := 150\frac{kN}{m^2}$ $\quad Y := 1.4m$

Given

$$\frac{Y \cdot p_{max} \cdot B_z}{2} = V_{y.b}$$

$$e_x + \frac{Y}{3} = \frac{B_x}{2}$$

$$vec := Find(Y, p_{max})$$

$$vec = \begin{pmatrix} 1.408m \\ 1.565 \times 10^5 Pa \end{pmatrix}$$

$$Y := vec_0$$

$$Y = 1.408m$$

$$p_{max} := vec_1$$

$$p_{max} = 156.483\frac{kN}{m^2}$$

B.3.3. STAAD Model

Electronic files:

Check3.std; Check3.xls

Results - Corner stresses:

Model 3

Node	L/C	Force-X kN	Force-Y kN	Force-Z kN
1	1	0.056	1.372	0
11	1	0.056	1.372	0
265	1	0.054	0	0
275	1	0.054	0	0

Support tributary area:

lx = 0.1 m ly = 0.09 m

A = 0.009 m^2

Soil pressures @ corners

Node	L/C	STAAD P (kN/m^2)	MATHCAD P (kN/m^2)	DIFFERENCE %
1	1	152.44	156.48	2.58
11	1	152.44	156.48	2.58
265	1	0.00	0	0.00
275	1	0.00	0	0.00

C. CONCLUSIONS

Comparing the results obtained from the STAAD.Pro analysis with those from the MATHCAD spreadsheet it is concluded that the difference between the results does not exceed five percent (5%) (The small values are not considered here because themselves do not represent 5 % of the maximum results). Although the 3D STAAD analysis may seem laborious and complicated, the use of the EXCEL spreadsheet for data input is decreasing the amount of work to a level comparable with a hand calculation for the foundations with net uplift.